MAT 1700

LØSNINGSFORSLAG

SEMINAR # 8

Løsningsforslag #8 Seminar

Oppgave 1

$$Q = 50L^{1/2}K^{1/2} \; ; \; \omega = 5 ; \; K = 20$$

$$\left.\begin{aligned} \frac{dQ}{dL} &= MP_L = 25L^{-1/2}K^{1/2} \\ \frac{dQ}{dK} &= MP_K = 25L^{1/2}K^{-1/2} \end{aligned}\right\} \quad \frac{MP_L}{MP_K} = L^{-1}K = \frac{K}{L}$$

Betingelse for 'optimum'

Kostnadsminimum $\Rightarrow \dfrac{MP_L}{MP_K} = \dfrac{K}{L} = \dfrac{\omega}{\uparrow} = \dfrac{5}{20} \; ; \; \underline{\underline{L = 4K}}$

og samtidig: $1000 = 50L^{1/2}K^{1/2}$

$(1000)^2 = 2500\, L \cdot K \; ; \quad \dfrac{1000{,}000}{2500} = L \cdot K \; ; \; \underline{\underline{L = \dfrac{400}{K}}}$

2 ligninger → 2 ukjente;

$$\begin{aligned} &L = 400/K \\ &\underline{-(L = 4K)} \end{aligned} \quad \Rightarrow \quad \frac{400}{K} = 4K \; ; \quad 100 = K^2$$

Kostnadsminimerende innsatskombinasjon $\begin{cases} \underline{\underline{10 = K}} \\ \underline{\underline{L = 4(10) = 40}} \end{cases}$

Oppgave 2

$$Q = 50L^{1/2}K^{1/2}$$

$$\frac{MP_L}{MP_K} = \frac{K}{L} = \frac{\omega}{r};$$

Betingelsen / "definisjonen" på 'optimum' (kostnads-minimum)

$$\Rightarrow L = \left(\frac{r}{\omega}\right)K$$

$$Q = 50\left[\frac{r}{\omega}K\right]^{1/2}K^{1/2}; \quad \frac{Q}{50} = \left(\frac{r}{\omega}\right)^{1/2} \cdot K$$

$$\frac{\frac{Q}{50}}{\left(\frac{r}{\omega}\right)^{1/2}} = K; \quad \boxed{\frac{Q}{50}\left(\frac{\omega}{r}\right)^{1/2} = \underline{\underline{K}}}$$

Since $L = \left(\frac{r}{\omega}\right)K = \frac{r}{\omega}\left[\frac{Q}{50}\left(\frac{\omega}{r}\right)^{1/2}\right]$

$$\underline{\underline{L}} = \frac{Q}{50}\left[\omega^{1/2} \times \omega^{-1} \times r \times r^{-1/2}\right] = \underline{\underline{\frac{Q}{50}\left(\frac{r}{\omega}\right)^{1/2}}}$$

Etterspørsel etter L og K forholder seg til prisen på innsatsfaktorene?

(i) L faller i ω, øker i r

(ii) K faller i r, øker i ω

K + L 'normal input'; ie. øker i Q

Oppgave 3

$Q = K^{1/2} L^{1/4} M^{1/4}$; $r = 2$, $\omega = 16$, $m = 1$

$MP_K = \frac{1}{2} K^{-1/2} L^{1/4} M^{1/4}$

$MP_L = \frac{1}{4} K^{1/2} L^{-3/4} M^{1/4}$

$MP_M = \frac{1}{4} K^{1/2} L^{1/4} M^{-3/4}$

(a) lang sikt; to "tangent-betingelser"

① $$\frac{MP_L}{MP_M} = \frac{\frac{1}{4} K^{1/2} L^{-3/4} M^{1/4}}{\frac{1}{4} K^{1/2} L^{1/4} M^{-3/4}} = \frac{M}{L} = \frac{\omega}{m} = \frac{16}{1}$$

② $$\frac{MP_L}{MP_K} = \frac{1/4 K^{1/2} L^{-3/4} M^{1/4}}{1/2 K^{-1/2} L^{1/4} M^{1/4}} = \frac{2}{4} K L^{-1} = \frac{\omega}{r} = \frac{16}{2}$$

$\frac{1}{2} K = 8L$; $K = 16L$

① $\underline{M = 16 \cdot L}$

② $\underline{K = 16 \cdot L}$ og i tillegg kreves at

③ $$Q = K^{1/2} L^{1/4} M^{1/4} = (16L)^{1/2} L^{1/4} (16L)^{1/4}$$

$$= 16^{(1/2 + 1/4)} L^{(1/2 + 1/4 + 1/4)} = 16^{3/4} L$$

$$= 16^{2/4} \cdot 16^{1/4} L = \sqrt{16} \times 2 \times L = \underline{\underline{8L}}$$

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Oppgave 3, con't

Så... når $Q = 8L$; $\underline{L = Q/8}$ og

$$m = 16L = 16\left(\frac{Q}{8}\right) = \underline{2Q}$$

$$K = 16L = 16\left(\frac{Q}{8}\right) = \underline{2Q}$$

"Long-run cost minimizing" input combination

(b) Short-run cost min. for $K = \bar{K}$ producing Q?

$$\frac{MP_L}{MP_m} = \frac{m}{L} = \frac{\omega}{m} = \frac{16}{1}\ ; \quad \underline{M = 16 \cdot L}\ ; \quad L = m/16$$

into $Q = \bar{K}^{1/2} L^{1/4} (16L)^{1/4} = \bar{K}^{1/2} L^{2/4}\, 16^{1/4}$

$$Q = \bar{K}^{1/2} L^{1/2} \cdot 2$$

$$Q^2 = 4L\bar{K}\ ; \quad \underline{L = \frac{Q^2}{4\bar{K}}} = \text{short-run cost-min of labor}$$

Solving for cost-min of M (materials):

$$Q = \bar{K}^{1/2}\left(\frac{m}{16}\right)^{1/4} m^{1/4} = \bar{K}^{1/2}\left(\frac{1}{16}\right)^{1/4} m^{1/4}\, m^{1/4}$$

$$= \bar{K}^{1/2} \times \frac{1}{2} \times m^{1/2}$$

$$Q^2 = \bar{K}\,\frac{1}{4}\,M\ ; \quad \underline{m = \frac{4Q^2}{\bar{K}}} = \text{short-run cost-min for materials}$$

Oppgave 3, con't

(c) $Q = 16$ og $\bar{K} = 32$ $\left(\Rightarrow K = 2Q = 2(16) = 32 \right)$

er etterspørsel etter L og M identisk på kort og lang sikt:

'Long-run' cost minimization $\begin{cases} L = Q/8 & \text{gives } L = 16/8 = \underline{\underline{2}} \\ M = 2Q & \text{"} \quad M = 2(16) = \underline{\underline{32}} \end{cases}$

'Short-run' cost minimization $\begin{cases} L = \dfrac{Q^2}{4K} = \dfrac{(16)^2}{4(32)} = \underline{\underline{2}} \\ M = \dfrac{4Q^2}{K} = \dfrac{4(16^2)}{32} = \underline{\underline{32}} \end{cases}$

Oppgave 4 (Ref. oppgave 2 her)

$$Q = 50 L^{1/2} K^{1/2}$$

From oppgave 2 above; $L = \dfrac{Q}{50}\left(\dfrac{r}{\omega}\right)^{1/2}$ og

$$K = \frac{Q}{50}\left(\frac{\omega}{r}\right)^{1/2}$$

"Kostnadsminimerende" total kostnad;

$$TC = \omega L + r \cdot K = \omega\left[\frac{Q}{50}\left(\frac{r}{\omega}\right)^{1/2}\right] + r\left[\frac{Q}{50}\left(\frac{\omega}{r}\right)^{1/2}\right]$$

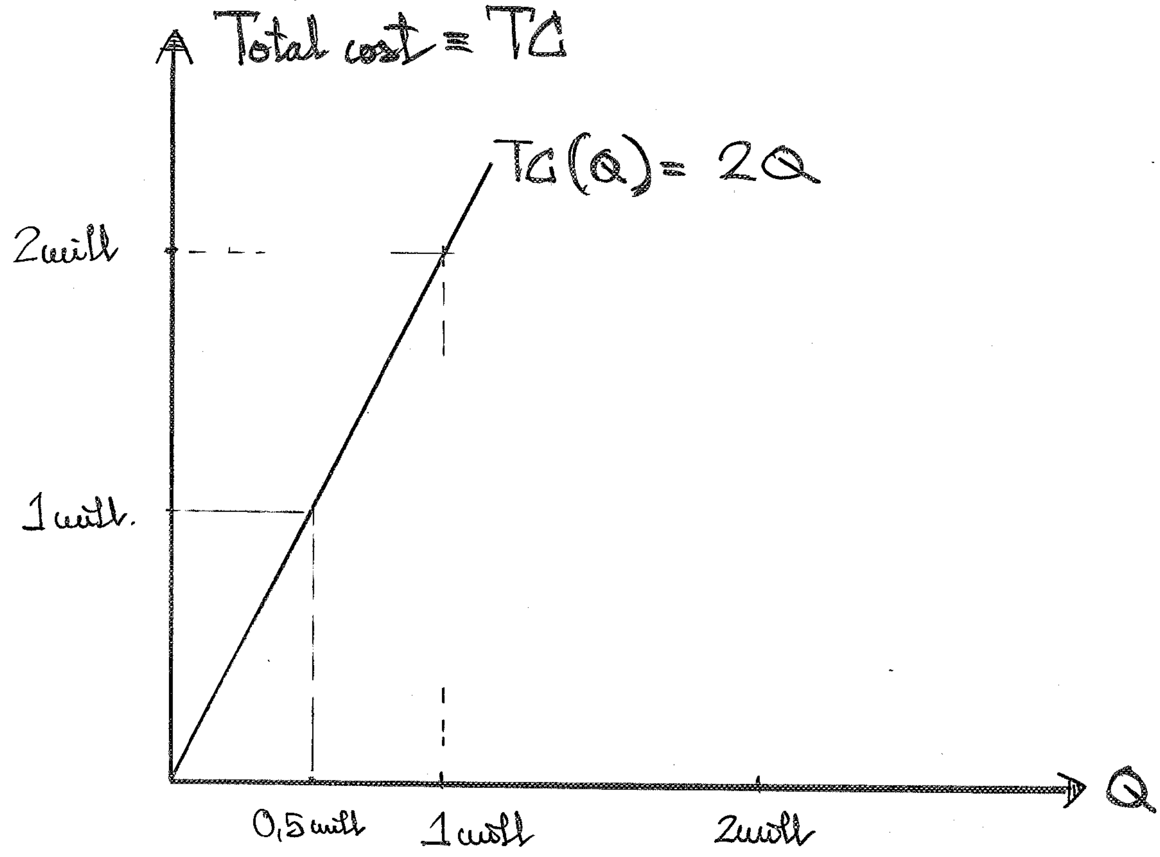
$$TC = \frac{2Q}{50}\left[\omega^{1/2} r^{1/2}\right] = \underline{\underline{\frac{Q}{25}\omega^{1/2} r^{1/2}}}$$

$$TC = \frac{2Q}{50}\left[\omega^{(1/2-1/2)} r^{1/2} \; r^{(1/2-1/2)} \cdot \omega^{1/2}\right] = \frac{2Q}{50}\left[\omega^{1/2} r^{1/2}\right]$$

Oppgave 4, con't

Long-run $TC \mid w = 25;\ r = 100$

$$TC = \frac{Q}{25}\left[25^{1/2}\ 100^{1/2}\right] = \frac{50Q}{25} = \underline{\underline{2Q}}$$



Oppgave 5 (Ref oppgave 3)

$$Q = K^{1/2} L^{1/4} M^{1/4} \quad ;$$

La $K = \bar{K}$; $w = \underline{16}$; $m = \underline{1}$ og $r = \underline{2}$

Short-run cost-min expressions for L and M:

$$L = Q^2/4\bar{K} \quad \text{og} \quad M = 4Q^2/\bar{K}$$

$$STC(Q) = wL + m \cdot M + r\bar{K}$$

$$= \frac{16Q^2}{4\bar{K}} + \frac{4Q^2}{\bar{K}} + 2\bar{K} = \underline{\underline{\frac{8Q^2}{\bar{K}} + 2\bar{K}}}$$

$$TVC(Q) = \underline{\underline{\frac{8Q^2}{\bar{K}}}} \quad = \text{total variable costs}$$

$$TFC(Q) = \underline{\underline{2\bar{K}}} \quad = \text{total fixed costs}$$